Model question in Statistics

Short answer type questions

1. In an election 10% of the voters on the voters' list did not cast their votes and 60 voters cast their ballot paper blank. There were only two candidates. The winner was supported by 47% of all voters in the list and he got 308 votes more than his rival. The number of voters on the list was (A) 3600 (B) 6200 (C) 4575 (D) 6028.

2. The digit in the unit position of the integer $1! + 2! + 3! + \ldots + 99!$ is:

   (a) 3
   (b) 0
   (c) 1
   (d) 7

3. The coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are 165,330 and 462. Then the value of n is :

   (a) 10
   (b) 12
   (c) 12
   (d) 11

4. The number of distinct positive integers that can be formed using 0,1,2,4 where each integer is used at the most once is equal to

   (a) 48
   (b) 84
   (c) 64
   (d) 36

5. Six numbers are in A.P. such that their sum is 3. The first number is four times the third number. The fifth number is equal to
   (A) -15; (B) -3; (C) 9; (D) -4.

6. Show that $\sec x = \frac{2}{\sqrt{2+\sqrt{2+2cos4x}}}$

7. $\sec\theta + \cos\theta = 2$. Find the value of $\sec^{17}\theta + \cos^{17}\theta$.

8. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1-\cos 2x}{2}}dx$ is

   (a) 1
   (b) 2

(c) $\frac{3}{2}$
(d) None of the above

9. A polygon has 35 diagonals. Then the number of sides is
   (a) 10
   (b) 9
   (c) 8
   (d) 7

10. If $0 \leq x \leq 1$ then find the minimum value of $x^2 + x + 1$.

11. If the equation of the locus of a point equidistant from the points $(a1, b1)$ and $(a2, b2)$ is $(a1 - a2)x + (b1 - b2)y + c = 0$, then the value of c is
    (a) $\frac{(a_2^2+b_2^2-a_1^2-b_1^2)}{2}$
    (b) $a_1^2 - a_2^2 + b_1^2 - b_2^2$
    (c) $\frac{a_1^2+a_2^2+b_1^2+b_2^2}{2})$
    (d) $\sqrt{a_1^2 - a_2^2 + b_1^2 - b_2^2}$

12. If $f(x)$ is a function which is both even and odd, then $f(3) - f(2)$ is equal to
    (a) 1
    (b) 1
    (c) 0
    (d) none

13. If $I_1 = \int_0^{\frac{\pi}{2}} \frac{x}{\sin(x)} dx$ and $I_2 = \int_0^1 \frac{\tan^{-1} x}{x} dx$; then $\frac{I_1}{I_2} =$
    (a) $\frac{1}{2}$
    (b) 2
    (c) 1
    (d) $\frac{\pi}{2}$

14. Does $\lim_{x\to n}(-1)^{[x]}$ exist , where n is an integer.

Broad question

15. Consider the function

$$f(x) = \lim_{n\to\infty} \frac{\log_e(2+x) - x^{2n}\sin(x)}{1+x^{2n}}$$

(a) Is $f(x)$ continuous at x=1? Justify your answer?

(b) ii) Show that $f(x)$ does not vanish anywhere in the interval $0 \leq x \leq \frac{\pi}{2}$; and indicate the points where $f(x)$ changes its sign.

16. At time 0, a particle is at the point 0 on the real line. At time 1, the particle divides into two and instantaneously after division; one particle moves 1 unit to the left and the other moves one unit to the right. At time 2, each of these particles divides into two, and one of the two new particles moves one unit to the left and the other moves one unit to the right. Whenever two particles meet, they destroy each other leaving nothing behind. How many particles will be there after time $2^{11} + 1$?

17. Given $n^4 < 10^n$ for a fixed positive integer n prove that $(n+1)^4 < 10^{n+1}$.

18. The line joining the points $A(b\cos\theta, b\sin\theta)$ and $B(a\cos\phi, b\cos\phi)$ is produced to the point $L(x, y)$ so that $AL : LB = b : a$. Then find the value of $x\cos\frac{\theta+\phi}{2} + y\sin\frac{\theta+\phi}{2}$.

19. Evaluate the series

$$\tfrac{1}{4\times 7} + \tfrac{1}{7\times 10} + \tfrac{1}{10\times 13} + \cdots + \infty$$

20. Find the limit of $\frac{\log(\cos(x))}{x^2}$ as $x \to \infty$.

21. Find all the pairs (x, y) of positive integers such that $x^2 + 3y$ and $y^2 + 3x$ are perfect squares.

22. Using the identity $\log x = \int_1^x dt/t, x > 0$, or otherwise, prove that

$$\tfrac{1}{n+1} \leq \log(1 + \tfrac{1}{n}) \leq \tfrac{1}{n} \text{ , for all integers } n \geq 1.$$