PUK 70STAT 03AT 130615 SAT 3-5 PM 100M

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[Attempt all 50 questions each bearing 2 marks. Each question has only one correct option]

1. The girder of a railway bridge is a parabola with its vertex at the highest point, 10 meters above the ends. If the span is 100 metres, find the height at 20 metres from the midpoint.

   (a) 1.0 meter (b) 6.4 meters (c) 8.4 meters (d) 3.2 meters

2. Let f(x) = x[x]. When x is not an integer, the value of f ′(x) is

   (a) 2x (b) [x] (c) 2[x] (d) It does not exists

3. What is the nature of the function $F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1}(x)$?

   (a) Non-decreasing (b) Non-increasing

   (c) Constant (d) First decreasing and then increasing

4. Suppose a point P(x, y) is moving in such a way that sum of distances of P from the points A (-2, 3) and B (2, 0) is constant equal to 4 units. The locus of P is

   (a) $(x-2)^2+(y+3)^2 = 5$ (b) $x^2+4y^2 = 16$ (c) $y^2 = 16x$ (d) Neither of these

5. Let $\Delta = \begin{vmatrix} 2 & 3 & 2 \\ x-u & y-v & z-w \\ 1 & 0 & 1 \end{vmatrix} = 0$. Then which of the following is correct?

   (a) $x = u, z = 2w$ (b) $y = u, z = 0$ (c) $y = v = x$ (d) $z - w = x - u$

6. $n^3 - n$ (n > 1 and is an integer) is always divisible by: (a) 6 (b) 7 (c) 3 (d) 9

7. Let P be an arbitrary point ~~lies~~ lying on the ellipse $9x^2+25y^2 = 225$. Then the sum of distances of the point from the foci is: (a)10 (b) 11 (c) 12 (d) 9

8. cos (5π /12) = ? (a) (√6 +√2)/3 (b) (√6 -√2)/3 (c) (√6 +√2)/4 (d) (√6 -√2)/4

9. The last digit of 307^{307} is: (a) 1 (b) 9 (c) 3 (d) 5

10. Let two circles of radius 5 cut each other at (1, 2). The equation of common tangent of these two circle is 4x+3y =10. Then the equations of the circles are

    (a) $x^2 + y^2 - 10x - 10y + 25 = 0$ *and* $x^2 + y^2 + 6x + 2y - 15 = 0$

    (b) $x^2 + y^2 = 25$ *and* $x^2 + y^2 = 16$

    (c) $x^2 + y^2 - 20x - 20y + 50 = 0$ *and* $x^2 - y^2 + 16 = 0$ (d) Neither of these

11. To find the value of the average of the three values *x*, *y* and *z*, which of the following is sufficient?

    (a) $x+y = 14$, if $x = y$ (b) $z - y = 3$, $x > y$ (c) $x = 5, y = z+2 = 3$ (d) $x = y = z$

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12. The value of $\int_{-\infty}^{\infty} \frac{1}{\pi} \frac{|x|}{1+x^2} dx$ is: (a) 0 (b) 1/2 (c) 1 (d) ∞

13. Let f(x) be a function such that f(f(x)) = x for all x $\in$ X. Then

(a) f(x) is one-to-one but need not be onto (b) f(x) is both one-to-one and onto

(c) f(x) is onto but need not be one-to-one (d) Neither of these statements are necessarily true

14. The value of $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{(n-i)(n+i)}{n^4}}$ is: (a) $\pi/2$ (b) $3\pi/4$ (c) $\pi/4$ (d) Neither of these

15. The maximum value attained by the function y = 10– |x – 10| is: (a) 10 (b) 9 (c) $+\infty$ (d) 1

16. Two persons *A* and *B* toss a coin 50 times each together. The probability that 'both of them get tails at the same times' is

(a) $(2^{50} - 51)/2^{100}$ (b) $50/2^{100}$ (c) $50^2/2^{50}$ (d) $(2^{50} - 1)/2^{100}$

17. Two buildings with flat roofs are 60m apart. From the roof of the shorter building, 40m in height, the angle of elevation to the edge of the roof of the taller building is 45^0. How high is the taller building? (a) 90m (b) 95m (c) 100m (d) 105m

18. Suppose the co-ordinates of three points A, B, C are (2, 1), (6, -2) and (8, 9) respectively. In Δ ABC, the inner equal bisector of $\angle$A has the equation

(a) x+y = 0 (b) x-7y+5 = 0 (c) x^2+3y = 6 (d) x+7y-5 = 0

19. In triangle ABC, 3 sin A + 4 cos B = 6 and 4 sin B + 3 cos A = 1. Then the measure of angle C is
(a) 30^0 (b) 45^0 (c) 60^0 (d)150^0

20. The probability that in a group of *N* (< 365) people, at least two will have the same birthday is

(a) $(N.\ 365!)/365^N$ (b) $(365!/[(365-N)!\ 365^N])$ (c) $(N/365)^{365}$ (d) ${}^{N}C_2 / {}^{365}C_N$

21. Let P(x) = $ax^2 + bx + c$, where a, b and c are real numbers. Also, P (2) > 0 and P(4) < 0.

Then a root of P(x) = 0, say, α must be: (a) 2< α <4 (b) α > 4 (c) 0< α <2 (d) α <0

22. $a^{2k+1} + b^{2k+1}$ (k is a positive integer) is divisible by

(a) $a^k \mp b^k$ (b) a + b (c) $a^k + b^k$ (d) a – b

23. If z = a + ib, a, b are real numbers, then |z/|z|| is: (a) $\frac{a+ib}{\sqrt{a^2+b^2}}$ (b) 1 (c) a + ib (d) $\frac{1}{\sqrt{a^2+b^2}}$

24. $(\log_b a)(\log_a b) =$? (a) $\log_b ab$ (b) $\log_{a+b} ab$ (c) $\log_{ab} a + b$ (d) Neither of these

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25. $(1-\cot 23^0)(1-\cot 22^0)$ =? (a) 1 (b) 2 (c) 3 (d) 4

26. If f(x) is a real valued differentiable function such that $f(x)f'(x) < 0$ for all real x, then it follows that

 (a) f(x) is an increasing function (b) f(x) is a decreasing function

 (c) |f(x)| is a decreasing function (d) |f(x)| is an increasing function

27. Which of the following value of n satisfies $(1+\tan 1^0)(1+\tan 2^0)\ldots\ldots(1+\tan 45^0) = 2^n$?

 (a) 23 (b) 24 (c) 25 (d)26

28. Six boys and six girls sit in a row randomly. The probability that 'all the girls sit together' is

 (a) 36/12! (b) 7/12 (c) (7! 6!)/12! (d) (6! 6!)/12!

29. If $A \subseteq B$, $B \subseteq C$, $C \subseteq A$ then which of the following is for sure?

 (a) $P(A \cup B \cup C) = 1$ (b) $P(A) + P(B) = 2\,P(C)$

 (c) $P(A) < P(B) < P(C)$ (d) Neither of these

30. The equation of the parabola with latus-rectum joining the points (2, 3) and (2, -5) is of the form: (a) $(x-\alpha)^2 = \pm 4(y-\beta)$ (b) $(x-\alpha)^2 = \pm 8(y-\beta)$ (c) $(y-\beta)^2 = \pm 16(x-\alpha)$ (d) $(y-\beta)^2 = \pm 8(x-\alpha)$

31. Which of the following are the solutions to the equation $\tan 2x + 2\sin x = 0$?

 (a) 0, $\pi/3$, π, $5\pi/3$ (b) 0, $\pi/2$, $3\pi/2$, $3\pi/5$ (c) 0, $\pi/4$, $5\pi/6$, $7\pi/8$ (d) Neither of these

32. In how many different ways 4 fruits can be distributed among 3 students so that each student gets at least one fruit? (a) 24 (b) 36 (c) 81 (d) 4

33. Let matrix $S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ and T be another matrix such that ST is an identity matrix of order 2. Then |T| is: (a) $-1/2$ (b) 1/2 (c) 1 (d) -1

34. The area of the region $\{(x, y): y > \max(-x, x)\}$ within the circle $x^2+y^2 = 1$ is

 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

35. $(4\cos^2 9^0 - 3)(4\cos^2 27^0 - 3)$ =? (a) $\tan 9^0$ (b) $\tan 10^0$ (c) $\tan 11^0$ (d) Neither of these

36. A function $f(x)$ of a variable x has a discontinuity point at $x = 2$. Which of the following can be said?

 (a) $f(x)$ is continuous for all $x < 2$ and for all $x > 2$ (b) $f(x) + 2$ also has the same discontinuity point

 (c) $[f(x)]^2$ is continuous for all x *(d) $f(x)$* cannot have any other discontinuity point

37. Which of the following statements is sufficient to judge whether x is an integer?

 (a) $\frac{x}{16}$ equals zero (b) $5x$ is an integer (c) $2x$ is a proper fraction (d) $7x+1$ is an integer

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38. An incident ray meets a straight line 3x-2y+7 = 0 along the path x-2y+5 = 0 and then is reflected. The equation of the reflected ray is
    (a) x-2y+5 = 0 (b) 4x-4y+12 = 0 (c) 15x+16y+32 = 0 (d) 29x-2y+33 = 0

39. In how many ways can one type the word *PRESIDENCY* such that only one letter is wrongly typed? (a) 250 (b) 520 (c) 2^{10} (d) $26^{10} - 10$

40. If $f(x) = \log_{x^2} e^x$, x >1, then the derivative f '(x) is: (a) $\frac{\log x+1}{(\log x)^2}$ (b) $\frac{\log x-1}{(\log x)^2}$ (c) $\frac{\log x+1}{2(\log x)^2}$ (d) $\frac{\log x-1}{2(\log x)^2}$

41. The locus of the point of intersection of two lines $\sqrt{3}x-y = 4\sqrt{3}\lambda$ and $\sqrt{3}\lambda x+ \lambda y = 4\sqrt{3}$, for any λ ($\neq 0$), is: (a) Parabola (b) Circle (c) Ellipse (d) Hyperbola

42. The value of $\sum_{k=1}^{n} k\ {}^{n}C_k$ is: (a) $n2^{n-1}$ (b) ${}^{2n+1}C_n$ (c) $n2^n$ (d) ${}^{2n-1}C_{n-1}$

43. Let f(x) be a non- negative continuous function such that f(x) + f (0.5 + x) = 1 for all x, $0 \leq x \leq 0.5$ then the value of $\int_0^1 f(x)dx$ is: (a) 0.25 (b) 0.5 (c) 1 (d) 2

44. The value of the integral $\int_{-1}^{2} [x]dx$ is: (a) 0 (b) 3 (c) 1 (d) 2

45. For four proper fractions *a, b, c, d*, ARJUN writes $a + b + c > 3(abc)^{1/3}$. IMRAN also added that $a + b + c > 3.(abcd)^{1/3}$. JOHN says that the above inequalities hold only if *a, b, c, d* are positive.
    (a) Both ARJUN and IMRAN are right but not JOHN (b) Only JOHN is right
    (c) Only ARJUN is right (d) Neither of these is absolutely right.

46. If we divide 3^{37} by 79, the remainder will be: (a) 2 (b) 11 (c) 7 (d) 35

47. Let $f(x) = \begin{cases} \left|\, |x-1| -1 \right|, & \text{if } x < 1 \\ [x], & \text{if } x \geq 1 \end{cases}$, where for any x, [x] denotes the largest integer $\leq$ x and |y| denotes the absolute value of y. Then the discontinuity points of the function f(x) are
    (a) all integers ≥ 0 (b) all integers ≥ 1 (c) all integers >1 (d) the integer 1.

48. Two numbers are chosen at a time from {1, 2, 3, 4, 5, 6}. The probability that 'at least one of these two is less than 4' is: (a) 4/5 (b) 1/15 (c) 1/5 (d) 3/5

49. The last digit of 17! will be: (a) 1 (b) 0 (c) 3 (d) 7

50. Find all x in the interval (0, $\pi/2$) such that [($\sqrt{3}$ - 1)/ sin x] + [($\sqrt{3}$ + 1)/ cos x] = $4\sqrt{2}$.
    (a) $\pi/9$, $2\pi/7$ (b) $\pi/36$, $11\pi/12$ (c) $\pi/12$, $11\pi/36$ (d) All